#### PHYSICAL ATTACKS (V1.3)

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### Agenda

Mathematical knowledge

Circuit knowledge

Cryptography knowledge

Attack methods

Case analysis

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# MATHEMATICAL KNOWLEDGE

Definition

A vector is *a list of numbers*. There are at least two ways to interpret what this list of numbers mean: One way to think of the vector as being appoint in a space. Then this list of numbers is a way of identifying that point in space, where each number represents the vector's component that dimension. Another way to think of a vector is magnitude and direction, e.g. a quantity like velocity.

#### • Vector addition and subtraction

Numerically, we add vectors component-by-component. That is to say, we add the x components together, and then separately we add the y components together.

$\vec{c} = \vec{a} + \vec{b}$	$\vec{c} = \vec{a} - \vec{b}$
$\vec{c} = [4,3] + [1,2]$	<sup>7</sup> [1 a] [1 a]
$\vec{c} = [4 + 1, 3 + 2]$	$\vec{c} = [4,3] - [1,2]$
$\vec{c} = [5,5]$	$\vec{c} = [3, 1]$

#### • Vector multiplication: dot product

A dot product(or scalar product) is the numerical product of the lengths of the two vectors, multiplied by the cosine of angle between them.

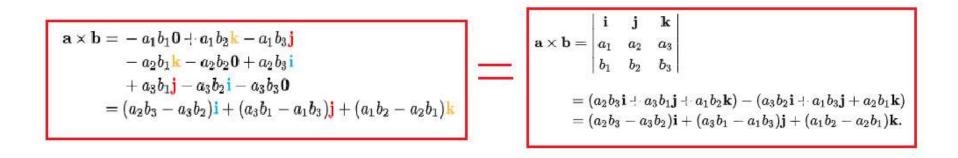
$$\vec{a} \cdot \vec{b} = [4,3] \cdot [1,2]$$
  
$$\vec{a} \cdot \vec{b} = (4*1) + (3*2)$$
  
$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$$
  
$$\vec{a} \cdot \vec{b} = 11$$

• Vector multiplication: cross product

A cross product of two vectors **a** and **b** is defined only in three-dimensional space and is denoted by **a x b**. The cross product is defined by the formula:

 $\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \sin(\theta) \mathbf{n}$ 

Where  $\theta$  is the angle between a and b in the plane containing them. ||a|| and ||b|| are the magnitudes of vector **a** and **b**, and **n** is a unit vector perpendicular to the plane containing a and b in the direction given by the right-hand rule.



### Determinant

#### Definition

A **square array** of numbers bordered on the left and right by a vertical line and having a value equal to the algebraic sum of all possible products where the number of factors in each product is the same as the number of rows or columns, each factor in a given product is taken from a different row and column, and sign of a product is positive or negative depending upon whether the number of permutations necessary to place the indices representing each factor's position in its row or column in the order of the natural numbers is odd or even.

### Determinant

• How to get the result of determinant.

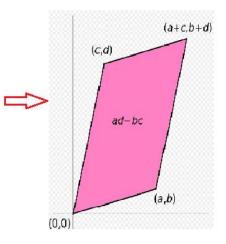
$$|A| = det(A) = \left|egin{array}{c} a & b \ c & d \end{array}
ight| = ad - bc$$

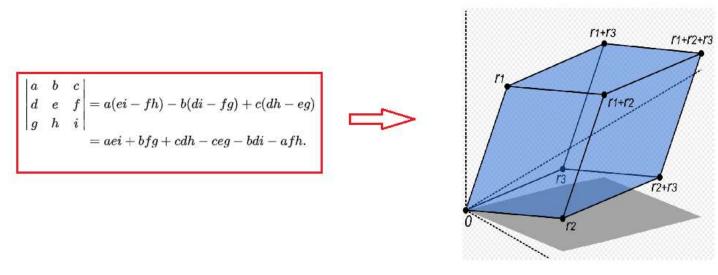
$$|B| = det(B) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = a \begin{vmatrix} e & f \\ h & k \end{vmatrix} - b \begin{vmatrix} d & f \\ g & k \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$
$$|A| = a_{11} (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21} (-1)^{2+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} (-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$
$$= a_{11} (a_{22} a_{33} - a_{23} a_{32}) - a_{21} (a_{12} a_{33} - a_{13} a_{32}) + a_{31} (a_{12} a_{23} - a_{13} a_{22})$$

#### Determinant

• Geometric meaning.

 $ext{Signed area} = |oldsymbol{a}||oldsymbol{b}|\sin heta = ig|oldsymbol{a}^{\perp}ig||oldsymbol{b}|\cos heta' = igg( rac{-b}{a}igg)\cdotigg( rac{c}{d}igg) = ad-bc.$ 





#### Definition

In mathematics, a matrix is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns. The numbers, symbols, or expressions in the matrix are called its entries or its elements. Matrix is commonly written in box brackets or parentheses.

Sy	mme	etric	]	Diag	jon:	al	Upper Triangular			Lower Triangular			Zero			Identity					
1	2	3]	[ 1	. (	כ	٥]	[ 1	2	3	ſ	1	0	٥]	ſ	0	0	٥]	ſ	1	0	0]
2	0	-5	0	4	4	0	0	7	-5		-4	7	0		0	0	0		0	1	0
3	-5	6	[ o		כ	6	L o	0	-4]		12	5	3		0	0	0		0	0	1

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m1} & \dots & a_{mn} \end{pmatrix}$$

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#### • Matrix addition

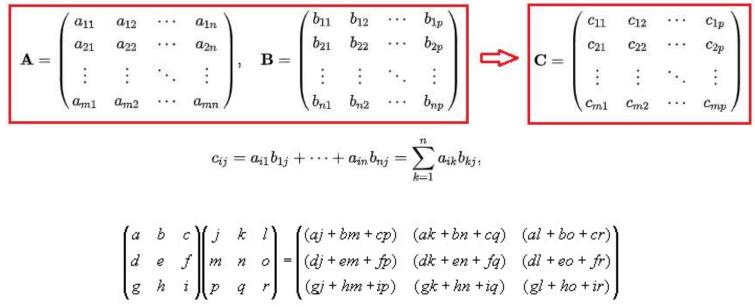
Two matrices **A** and **B** can be added or subtracted if and only if their dimensions are the same(both matrices have the equal numbers of rows and columns). The sum of two matrices **A** and **B** will be a matrix which has the same number of rows and columns as do **A** and **B**.

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$
$$= \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{bmatrix}$$

#### Matrix multiplication

In mathematics, matrix multiplication or matrix product is a binary operation that produces a matrix from two matrices with entries in a filed, or, more generally, in a ring or even a semiring.

If **A** is an  $m \times n$  matrix and **B** is an n x p matrix, the matrix product **C** = **AB** is defined to be the  $m \times p$  matrix.



#### • Matrix transposition

In linear algebra, the transpose of a matrix is an operator which flips a matrix over its diagonal, that is switches the row and column indices of the matrix by producing another matrix denoted. Some properties of transpose:

$$\begin{split} \left(\mathbf{A}^{\mathrm{T}}\right)^{\mathrm{T}} &= \mathbf{A}. \\ \left(\mathbf{A} + \mathbf{B}\right)^{\mathrm{T}} &= \mathbf{A}^{\mathrm{T}} + \mathbf{B}^{\mathrm{T}}. \\ \left(\mathbf{A}\mathbf{B}\right)^{\mathrm{T}} &= \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}. \\ \left(c\mathbf{A}\right)^{\mathrm{T}} &= c\mathbf{A}^{\mathrm{T}}. \\ \det\left(\mathbf{A}^{\mathrm{T}}\right) &= \det(\mathbf{A}). \\ \left[\mathbf{a} \cdot \mathbf{b}\right] &= \mathbf{a}^{\mathrm{T}}\mathbf{b}, \end{split}$$

$$A = \begin{pmatrix} 5 & 2 & 3 \\ 4 & 7 & 1 \\ 8 & 5 & 9 \end{pmatrix} \quad A^T = \begin{pmatrix} 5 & 4 & 8 \\ 2 & 7 & 5 \\ 3 & 1 & 9 \end{pmatrix}$$

The dot product of two column vectors a and b can be computed as the single entry of the matrix product.  $(\mathbf{A}^{\mathrm{T}})^{-1} = (\mathbf{A}^{-1})^{\mathrm{T}}.$ 

#### • Invertible matrix

In linear algebra, an n-by-n square matrix A is called invertible if there exists an n-by-n square matrix B such that  $AB = BA = I_n$ , where  $I_n$  denotes the n-by-n identity matrix and the multiplication used is ordinary matrix multiplication.

If A is an n x n invertible matrix , then  $A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$ .

Where adj(A) is the transpose of the cofactor matrix **C** of **A**.

#### • Rank

In linear algebra, the rank of a matrix **A** is the dimension of the vector space generated(or spanned) by its columns. **rank(A)**.

 $\operatorname{rank}(A) = \operatorname{rank}(A^T)$ 

• Eigenvalues and Eigenvectors

Let **A** be a square matrix. A non-zero column vector  $\boldsymbol{v}$  is called an eigenvector if  $A\boldsymbol{v}$  is parallel to v, i.e., if there exists a scalar  $\boldsymbol{\lambda}$  such that  $A\boldsymbol{v} = \boldsymbol{\lambda}\boldsymbol{v}$ 

The scalar  $\boldsymbol{\lambda}$  is then called an eigenvalue.

#### Example 10.18: Eigenfaces

The main idea is that one can *learn* how images of faces look like and use this information for *image* or *video compression*, for *face detection*, or *face recognition*. Briefly, this is done by first calculating the mean and the covariance matrix for a set of images. Then the *eigenvectors* and *eigenvalues* of the covariance matrix are calculated. The eigenvectors corresponding to the largest eigenvalues are the ones that best encode image changes. This is an example of *machine learning*. In more general terms, *machine learning* are tools for learning good representations of data, or learning how to recognize images. Methods involving linear algebra, mathematical statistics, analysis, and optimization are used on (often large) collections of data.

### Mean(expected value)

#### Definition

In probability theory, the expected value of a random variable is a key aspect of its probability distribution. Intuitively, a random variable's expected value represents the average of a large number of independent realizations of the random variable. It is often represented  $\mu, \overline{X}$ .

Let X be a random variable with a finite number of finite outcomes x1, x2,  $\cdots$  xk occurring with probabilities p1, p2,  $\cdots$  pk. The expectation of X is defined as :

$$\mathrm{E}[X]=\sum_{i=1}^k x_i\,p_i=x_1p_1+x_2p_2+\cdots+x_kp_k.$$

Where all probabilities pi add up to  $1 (p1 + p2 + \dots + pk = 1)$ .

If all outcomes xi are equiprobable, then the expected value is average(arithmetic mean):

$$A=rac{1}{n}\sum_{i=1}^n a_i=rac{a_1+a_2+\dots+a_n}{n}$$

### Variance

#### Definition

In probability theory and statistics, variance is the expectation of the squared deviation of a random variable from its mean. Informally, it measures how far a set of (random) numbers are spread out from their average value. The variance is the square of the standard deviation. It is often represented by  $\sigma^2$ ,  $s^2$ , or Var(X).

The variance of a random variable X is the expected value of the squared deviation from the mean of X,  $\mu = E[X]$ :

 $\operatorname{Var}(X) = \operatorname{E}[(X - \mu)^2]$ 

### Covariance

 In probability theory and statistics, covariance is a measure of the joint variability of two random variables. If the greater values of one variable mainly correspond with the greater values of the other variable, and the same holds for the lesser values, (i.e., the variables tend to show similar behavior), the covariance is positive. In the opposite case, when the greater values of one variable mainly correspond to the lesser values of the other, the covariance is negative. The sign of the covariance shows the tendency in the linear relationship between the variables.

$$\mathrm{cov}(X,Y) = \mathrm{E}\left[(X-\mathrm{E}[X])(Y-\mathrm{E}[Y])
ight]$$

$$\operatorname{cov}(X,Y) = rac{1}{n}\sum_{i=1}^n (x_i - E(X))(y_i - E(Y))$$

# Normal distribution

Definition

Normal distribution, also known as the Gaussian distribution, is a probability and is that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data far from the mean.

The probability density of the normal distribution is(PDF)

$$f(x\mid \mu,\sigma^2)=rac{1}{\sqrt{2\pi\sigma^2}}e^{-rac{(x-\mu)^2}{2\sigma^2}}$$

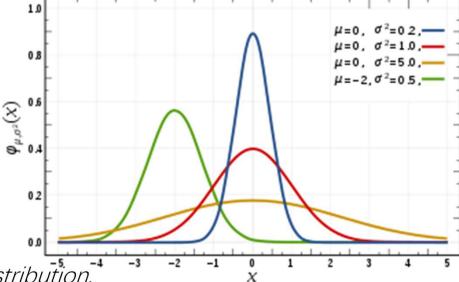
Whe. \_

 $\mu$  is the mean or expectation of the distribution

 $\sigma^2$  is the variance

when  $\mu = 0$  and  $\sigma = 1$ , called as standard normal distribution.



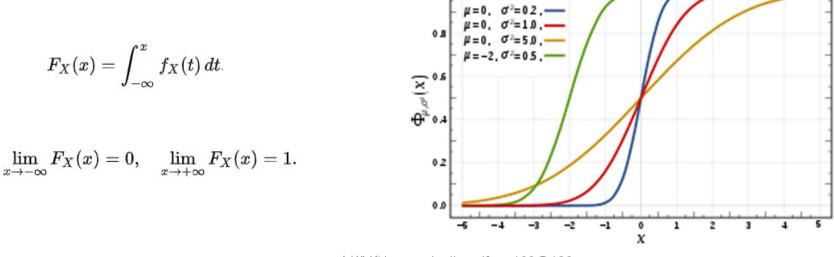


Normal distribution

• The normal distribution is often revered to as  $N(\mu, \sigma^2)$ , Thus when a random variable X is distributed normally with mean  $\mu$  and variance  $\sigma^2$ , one may write

 $X \sim N(\mu, \sigma^2)$ 

• The cumulative distribution function(CDF) of a real-valued random variable X, or just distribution function of X, evaluated at x, is the probability that X will take a value less than or equal to x.



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## Multivariate distribution Statistics

- Multivariate distributions let us model multiple random variables that may or may not be correlated.
- A whole matrix of covariances. E.g three random variables(X,Y,Z).

• A mean for each random variable:  

$$\mu = \begin{bmatrix} \mu X \\ \mu Y \\ \mu Z \end{bmatrix} \qquad \Sigma = \begin{bmatrix} Var(\mathbf{X}) & Cov(\mathbf{X}, \mathbf{Y}) & Cov(\mathbf{X}, \mathbf{Z}) \\ Cov(\mathbf{Y}, \mathbf{X}) & Var(\mathbf{Y}) & Cov(\mathbf{Y}, \mathbf{Z}) \\ Cov(\mathbf{Z}, \mathbf{X}) & Cov(\mathbf{Z}, \mathbf{Y}) & Var(\mathbf{Z}) \end{bmatrix}$$

• About the PDF, it uses a vector with all of the variables  $x = [x, y, z, ...]^T$ , The equation for k random variables is

$$f(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^k |\mathbf{\Sigma}|}} e^{-((\mathbf{x}-\mu)^T \mathbf{\Sigma}^{-1} (\mathbf{x}-\mu)/2}$$

• The SciPy package in Python can do all for this.

#### Welch's t-test

- In statistics, welch's t-test, or unequal variances t-test, is a two-sample location test which is used to test the hypothesis that two populations have equal means.
- Welch's t-test defines the  $t = \frac{\overline{X}_1 \overline{X}_2}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}$  following formula:

Where  $\overline{X1}$ ,  $s_1^2$  and  $N_1$  are the 1<sup>st</sup> sample mean, sample variance and sample size.

### Finite field

- In mathematics, a finite field or Galois field is a field that contains a finite number of elements.
- The number of elements of finite field is called its *order*, or size.
- A finite field of order q exists if and only if the order q is a prime power  $p^k$  (where p is a prime number and k is a positive integer).

 $\begin{array}{l} F_2 \mbox{ or } GF(2) \mbox{ is } [0,1] \\ F_{11} \mbox{ or } GF(11) \mbox{ is } [0,1,2,3,4,5,6,7,8,9,10,11] \\ F_{23} \mbox{ or } GF(23) \mbox{ is } [0,1,2,3,4,5,6,7,8,9,10,11,12...22] \end{array}$ 

# Chinese remainder theorem

• If the  $n_i$  are pairwise coprime, and if  $a_1, ..., a_k$  are any integers, then there exists an integer x such that  $x \equiv a_1 \pmod{n_1}$ 

$$x \equiv a_{1} \pmod{n_{1}}$$

$$x \equiv a_{k} \pmod{n_{k}}$$

$$x = \left(\sum_{i=1}^{k} x_{i}r_{i}s_{i}\right) \mod n$$

$$= (x_{1}r_{1}s_{1} + x_{2}r_{2}s_{2} + \dots + x_{k}r_{k}s_{k}) \mod n$$

$$n = n_1 n_2 \dots n_k,$$
  

$$r_i = \frac{n}{n_i},$$
  

$$s_i = r_i^{-1} \mod n_i.$$

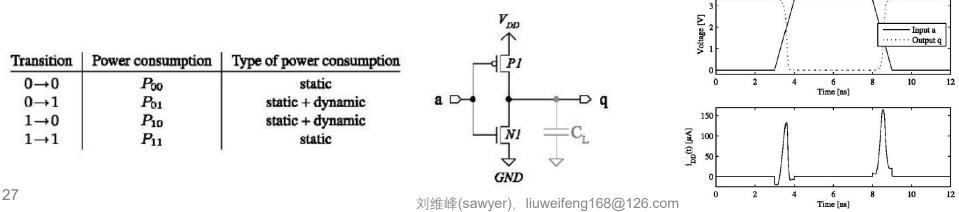
Note: a and b are coprime if and only if gcd{a, b} = 1.

$x \equiv 3 \pmod{5}$	bi	$N_i = \frac{N}{N_i}$	Xi	b:N:x:	$N = n_1 n_2 n_3$ $N_i = \frac{N_i}{n_i}$
$x \equiv 1 \pmod{7}$ $x \equiv 6 \pmod{8}$	61 62 63	$N_{1} = n_{1}n_{3}$ $N_{2} = n_{1}n_{5}$ $N_{3} = n_{1}n_{2}$	x, x z 3	$b_1 N_1 \times 1$ $b_2 N_2 \times 2$ $b_3 N_3 \times 3$	$x = \underbrace{\underset{i=1}{\overset{3}{\underset{i=1}{{\atopi}{\underset{i=1}{{\underset{i=1}{{\underset{i=1}{{\underset{i=1}{{\underset{i=1}{{\underset{i=1}{{\atopi}{\atopi}{\atopi}{\atopi}}}}}}}}}}}}}}}}}}}}$
Remainders			¢1	nverse of N	li
N-C7 0	bi	Ni	Xi	beNixe	
N=5x7x8 =280	3 1 6	56 40 35	-mm	168 120 630	
Check: 78 = 3 ( 78 = 1 78 = 6	x = x =	168+12( 918 (mod 78 (mod	280	30 = 918 )	

# CIRCUIT KNOWLEDGE

# Power Consumption of CMOS Circuits

- Power consumption is divided into two parts: static power consumption  $P_{stat}$  and dynamic power consumption  $P_{dyn}$ .
- The static power consumption is calculated  $P_{stat} = I_{leak} \cdot V_{DD}$ .
- The static power consumption of CMOS circuits is typically very low.
- Dynamic power consumption occurs if an internal signal or an output signal of a logic cell switches.
- The dynamic power consumption depends on the data that is processed by the CMOS circuit.



# Hamming weight

• Hamming weight: the number of non-zero symbols in a symbol sequence. Note: for binary signaling, hamming weight is the number of "1" bits in the binary sequence.

String	Hamming weight
11101	4
11101000	4
00000000	0
789012340567	10

# Hamming Distance

- In information theory, the hamming distance between two strings of equal length is the number of positions at which the corresponding symbols are different.
- For binary strings *a* and *b* the hamming distance is equal to the number of ones in *a* XOR *b*.

HD(v0, v1) = HW(v0 XOR v1)

- Hamming distance model is very well suited to describe the power consumption of data buses.
- Attackers commonly use the hamming distance model to describe the power consumption of buses and registers.

#### **Noise distributions**

• Electrical signals are inherently noisy.

$$X = X_{actual} + N$$

• The probability density function(PDF) of Gaussian distribution is

 $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$ , where  $\mu$  is the mean and  $\sigma$  is the standard deviation. python: stats.norm.pdf(x, $\mu$ , $\sigma$ )

• Find probability using the CDF(cumulative distribution function).

```
mu = 5
sigma = 0.5
from scipy import stats
pval = stats.norm.cdf(6.0, mu, sigma)
print("%.2f %%"%(pval*100))
97.72 %
```

#### SNR

• The "Signal to Noise Radio" is defined as:

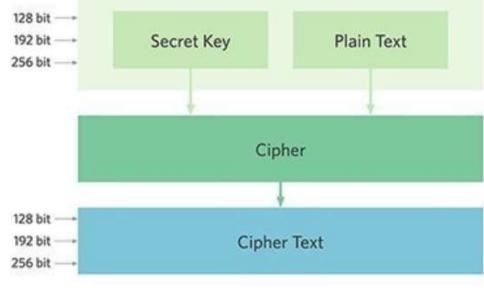
 $SNR = \frac{Var(Signal)}{Var(Noise)}$ 

when  $\mu = 0$ .

# CRYPTOGRAPHY KNOWLEDGE

#### AES

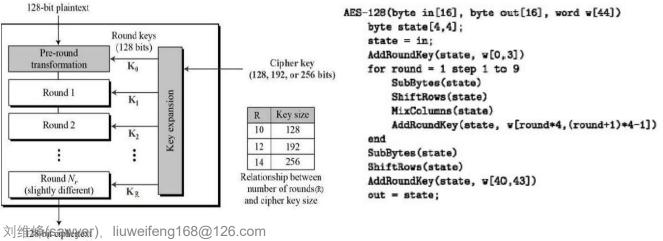
- The advanced encryption standard, or AES, is a symmetric block cipher chosen by the U.S. government to protect classified information and is implemented in software and hardware throughout the world to encrypt sensitive data.
- AES comprises three block ciphers: AES-128, AES-192 and AES-256. Each cipher encrypts and decrypt data in blocks of 128 bits using cryptographic key of 128-, 192- and 256-bits.



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### AES

- High-level description of the algorithm, see <u>https://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.197.pdf</u>
  - 1. key expansion round keys are derived from the cipher key using <u>Rijndael's key schedule</u>.
  - 2. initial round key addition:
  - 1 AddRoundKey each byte of the state is combined with a block of the round key using bitwise xor.
  - 3. 9, 11 or 13 rounds:
    - ① Subbytes—a non-linear substitution step where each byte is replaces with another according to lookup table.
    - 2 ShiftRows– a transposition step where the last three rows of the states are shifted cyclically a certain number of steps.
    - ③ Mixcolumns- a linear mixing operation which operates on the columns of the state, combining the four bytes in each column.
      128-bit plaintext
      AES-128 (byte in[16], byte out[16], word w[44]
    - 4 AddRoundKey
  - 4. Final round(making 10, 12 or 1<sup>2</sup> rounds in total):
    - ① subBytes
    - 2 shiftRows
    - ③ AddRoundKey



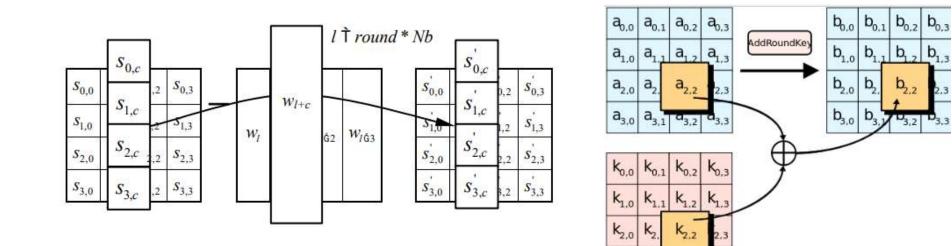
#### AES

• KeyExpansion(key Schedule)

```
KeyExpansion(byte key[4*Nk], word w[Nb*(Nr+1)], Nk)
      begin
                          w[44]/w[52]/w[60] as output
         word temp
                          key[16]/key[24]/key[32] as input
         i = 0
         while (i < Nk)
            w[i] = word(key[4*i], key[4*i+1], key[4*i+2], key[4*i+3])
            i = i + 1
         end while
         i = Nk
         while (i < Nb * (Nr+1)]
            temp = w[i-1]
            if (i mod Nk = 0)
               temp = SubWord(RotWord(temp)) xor Rcon[i/Nk]
            else if (Nk > 6 and i mod Nk = 4)
               temp = SubWord(temp)
            end if
            w[i] = w[i-Nk] xor temp
            i = i + 1
         end while
      end
      Note: Nk = 4, or 6, or 8.
            Nb: Nb words of key data for each round, Nb = 4 for AES.
            Nr: number of ounds, Nr = 10, 12 or 14
 SubWord(): SubWord([b_0 \ b_1 \ b_2 \ b_3]) = [S(b_0) \ S(b_1) \ S(b_2) \ S(b_3)]
 RotWord(): RotWord([b_0 \ b_1 \ b_2 \ b_3]) = [b_1 \ b_2 \ b_3 \ b_0]
    Rcon[]: rcon_i = [rc_i \quad 00_{16} \quad 00_{16} \quad 00_{16}]
```

## AES

AddRoundKey



( k<sub>3,1</sub>)

3,2

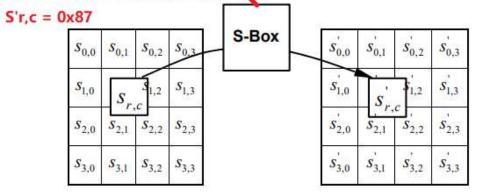
k<sub>3,0</sub>

## AES

• Subbytes

									3	7							
		0	1	2	3	4	5	6	7	8	9	a	b	C	d	е	f
	0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
	1	ca	82	c9	7d	fa	59	47	fO	ad	d4	a2	af	9c	a4	72	c0
	2	b7	fd	93	26	36	3f	£7	cc	34	a5	e5	f1	71	d8	31	15
	3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
	4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
	5	53	d1	00	ed	20	fc	<b>b1</b>	5b	6a	cb	be	39	4a	4c	58	cf
	6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
	7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
x	8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
	9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
	a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
	b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
	C	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
	d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
	е	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
	f	8c	al	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

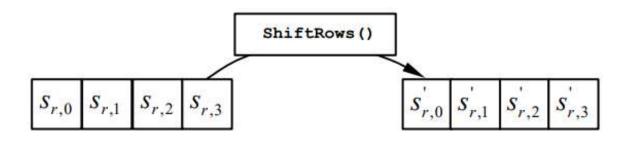
e.g. Sr,c = 0xEA, x = 0xE, y = 0xA



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• Shiftrows





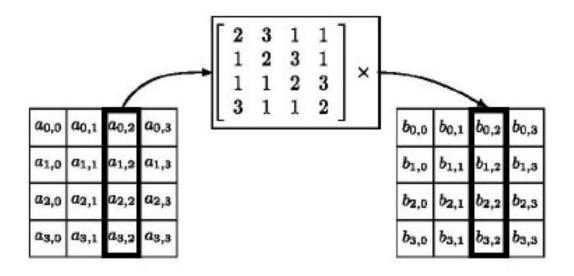


<i>S</i> <sub>0,0</sub>	<i>S</i> <sub>0,1</sub>	<i>S</i> <sub>0,2</sub>	<i>S</i> <sub>0,3</sub>		<i>S</i> <sub>0,0</sub>	<i>S</i> <sub>0,1</sub>	<i>S</i> <sub>0,2</sub>	<i>S</i> <sub>0,3</sub>
<i>S</i> <sub>1,0</sub>	<i>s</i> <sub>1,1</sub>	<i>s</i> <sub>1,2</sub>	<i>s</i> <sub>1,3</sub>		<i>S</i> <sub>1,1</sub>	<i>s</i> <sub>1,2</sub>	<i>s</i> <sub>1,3</sub>	<i>S</i> <sub>1,0</sub>
<i>S</i> <sub>2,0</sub>	<i>s</i> <sub>2,1</sub>	<i>s</i> <sub>2,2</sub>	<i>S</i> <sub>2,3</sub>	┌────◄	<i>s</i> <sub>2,2</sub>	<i>S</i> <sub>2,3</sub>	<i>S</i> <sub>2,0</sub>	<i>s</i> <sub>2,1</sub>
S <sub>3,0</sub>	<i>S</i> <sub>3,1</sub>	<i>S</i> <sub>3,2</sub>	<i>S</i> <sub>3,3</sub>	┌────◄	<i>s</i> <sub>3,3</sub>	<i>S</i> <sub>3,0</sub>	<i>S</i> <sub>3,1</sub>	<i>S</i> <sub>3,2</sub>

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## AES

#### • Mixcolumns



b0,0 = 2a0,0 + 3a1,0 + a2,0 + a3,0

## RSA

- 1. Generate two large random primes, p and q, of approximately equal size such their product n = pq is of the required bit length, e.g. 1024 bits.
- 2. Compute n = pq and  $\emptyset = (p 1)(q 1)$
- 3. Choose an integer e,  $1 < e < \emptyset$ , such  $gcd(e, \emptyset) = 1$ . Compute the secret exponent d,  $1 < d < \emptyset$ , such that  $ed \equiv 1 \mod \emptyset$ .
- 4. The **public key is (n, e)** and the private key (d, p, q). Sometimes the **private key is written as (n, d)**
- *n is known as the modulus, bit length of n is 1024, 2048, 3072, 4096.*
- *e is known as the public exponent or encryption exponent or just the exponent. e = 3,5,17,257,65537.*
- *d is known as the secret exponent or decryption exponent.*

## RSA

- Encrypting message
  - The message **M** is turned into a number *m* small than n by using an agree-upon reversible protocol known as a <u>padding scheme</u>(such as PKCS).
  - Compute the ciphertext **c**:
  - $c = m^e \mod n$
- Decrypting message

 $m = c^d \mod n$ 

## RSA

- Signature
  - The massage  ${\sf M}$  is digested with digest algorithm to give an octet string  ${\sf MD}.$
  - Data encoding. MD and digest algorithm identifier are combined into an ASN.1 value, which shall be BER-encoded to give an octet string D.
  - The data D is encrypted with RSA private key( $s = m^d \mod n$ ) to give an octet string ED.
  - The ED is converted into a bit string S, the signature.
- Verification
  - Signature S is converted into an octet string ED.
  - The ED is decrypted with RSA public key( $c = m^e \mod n$ ) to give an octet string D.
  - The D is BER-decoded to give an ASN.1 value type DigestInfo(message digest MD and message-digest algorithm identifier).
  - The massage M is digested with the selected message-digest algorithm to MD'.
  - If MD' is the same as the MD, the verification shall be succeeded.

## ECC

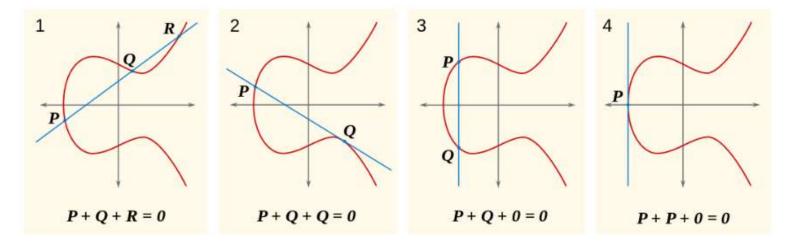
- ECC(Elliptic-curve cryptography) is an approach to public-key cryptography based on the algebraic structure of elliptic curves over finite fields.
- Elliptic curves are applicable for key agreement, digital signatures, pseudo-random generators and other task.
- The primary benefit promised by elliptic curve cryptography is a smaller key size. A 256-bit elliptic curve public key should provide comparable security to 3072-bit RSA public key.

## ECC

#### • Elliptic curve

In mathematics, an elliptic curve is a plane algebraic curve defined by an equation of the form  $y^2 = x^3 + ax + b$ 

Where **a** and **b** are real numbers. This type of equation is called a Weierstrass equation. Algebraically, this holds if and only if the discriminant  $-16(4a^3 + 27b^2) \neq 0$ . In finite filed( $F_p$ ), If P + Q + R = 0, Then  $P + Q \equiv R(mod p)$ ,  $P \times Q \equiv R(mod p)$ ,  $P \div Q \equiv R(mod p)$ .



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## ECC

- Implementation
  - 1. Define a base point G in the elliptic curve, the order of G, that is the smallest positive number *n* such that nG = 0,
  - 2. Since n is the size of a subgroup of E[Fp] it follows from Lagrange's theorem that the number  $h = \frac{1}{n} |E(Fp)|$  is an integer.
  - 3. In the prime case, the domain parameters are *(p, a, b, G, n, h).*
  - 4. Key pair, K = kG(k < n), K is public key, k is private key.
  - 5. e.g. parameters secp256k1(http://www.secg.org/sec2-v2.pdf)

#### secp256k1

$$= 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1$$

The curve E:  $y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

- The base point G in compressed form is:
  - G = 02.79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9 59F2815B 16F81798

and in uncompressed form is:

G = 04 79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9 59F2815B 16F81798 483ADA77 26A3C465 5DA4FBFC 0E1108A8 FD17B448 A6855419 9C47D08F FB10D4B8

Finally the order n of G and the cofactor are:

- h = 01

# ATTACK METHODS

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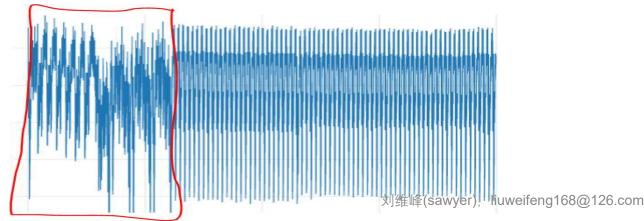
#### **SPA--definition**

• Simple power analysis (SPA) is a side-channel attack which involves visual examination of graphs of the current used by a device over time. Variations in power consumption occur as the device performs different operations. For example, different instructions performed by a microprocessor will have differing power consumption profiles. As a result, in a power trace from a smart card using DES encryption, the sixteen rounds can be seen clearly. Similarly, squaring and multiplication operations in RSA implementations can often be distinguished, enabling an adversary to compute the secret key. Even if the magnitude of the variations in power consumption are small, standard digital oscilloscopes can easily show the data-induced variations. Frequency filters and averaging functions (such as those built into oscilloscopes) are often used to filter out high-frequency components.

#### SPA—example(password check)

- Input password via UART, then check if the password inputted is correct.
- Attack the full password(keep guessing letters until we no longer see the distinctive spike).
- Input password guess and capture trace "cap\_pass\_trace".
- Analyze trace(checkpass function)

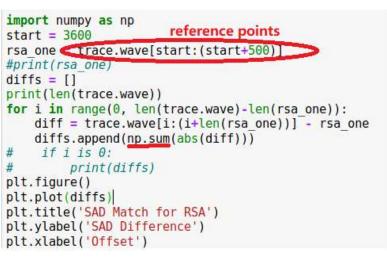
48

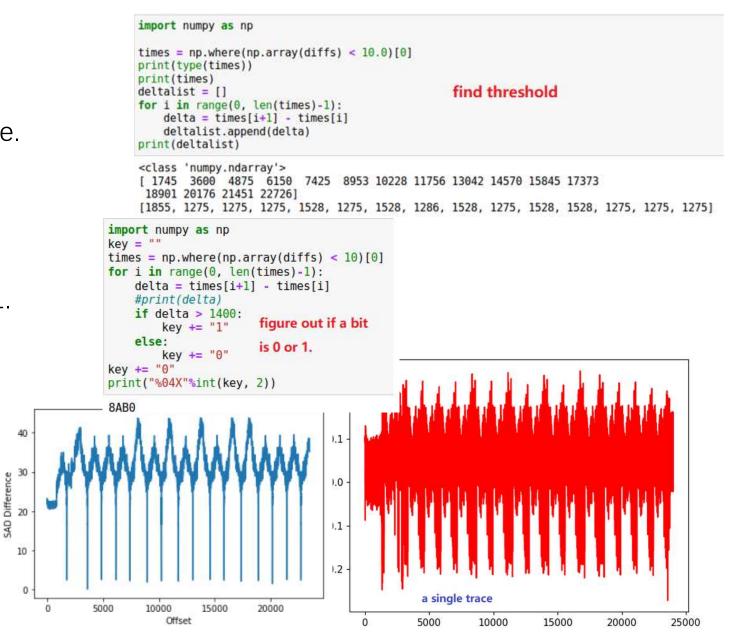




#### SPA-RSA

- Capture a single RSA trace.
- Select "reference" points.
- SAD match for RSA.
- Find a good threshold.
- Figure out if a bit is 0 or 1.



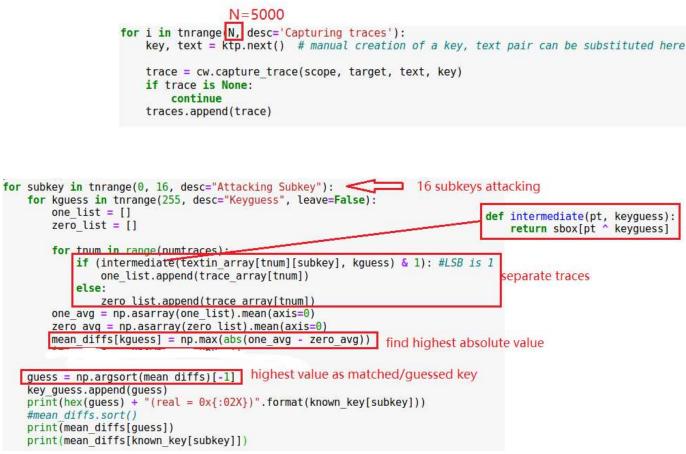


#### **DPA**—definition

 Differential power analysis (DPA) is a <u>side-channel attack</u> which involves statistically analyzing power consumption measurements from a <u>cryptosystem</u>. The attack exploits biases varying power consumption of microprocessors or other hardware while performing operations using secret keys. DPA attacks have *signal processing* and *error correction* properties which can extract secrets from measurements which contain too much noise to be analyzed using simple power analysis. Using DPA, an adversary can obtain secret keys by analyzing power consumption measurements from multiple cryptographic operations performed by a vulnerable smart card or other device.

### **DPA**—example(AES DPA Attack)

- Capture traces(5000 traces).
- Separate traces based on the least significant bit.
- Calculate the difference of means, the highest absolute value corresponds to the guessed sub key.
- Repeat this with each possible key guess.



#### **CPA**—definition

- Correlation Power Analysis(CPA) is an attack that allows us to find a secret encryption key that is stored on a victim device.
- There are 4 steps to a CPA attack:
  - Write down a model for the victim's power consumption.
  - Get the victim to encrypt several different plaintexts. Record a trace of the victim's power consumption during each of these encryptions.
  - Attack small parts(subkeys) of the secret key:
  - Consider every possible option for the subkey. For each guess and each trace, use the known plaintext and the guessed subkey to calculate the power consumption according to our model.
  - Calculate the pearson correlation coefficient between the modeled and actual power consumption.
  - Decide which subkey guess correlates best to the measured traces.
  - Put together the best subkey guesses to obtain the full secret key.

#### **CPA**—modeling power consumption

- One of the simplest models for power consumption is the Hamming Distance.
  - -e.g HammingDistance(00110000, 00100011) = 3
  - HammingDistance(x, y) = HammingWeight(x ^ y)

#### **CPA**—pearson's correlation coefficient

• Pearson's correlation coefficient:

 $\rho_{X,Y} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sqrt{E[(X - \mu_X)^2]E[(Y - \mu_Y)^2]}}$ 

This correlation coefficient will always be in the range[-1, 1], it describes how closely the random variables X and Y are related:

If Y always increases when X increases, it will be 1;

If Y always decreases when X increases, it will be -1;

If Y is totally independent of X, it will be 0.

#### **CPA**—attack with correlation

- After taking our measurements, we'll have D power traces, and each trace will have T data points.  $t_{d,j}$  will refer to point j in the trace  $d(1 \le d \le D, 0 \le j \le T)$
- There are I different subkeys that we want to try. hai will refer to our power estimate in trace d.
- Calculate this is(how well model and measurements match for each guess i and time j).

$$r_{i,j} = \frac{\sum_{d=1}^{D} \left[ \left( h_{d,i} - \overline{h_i} \right) \left( t_{d,j} - \overline{t_j} \right) \right]}{\sqrt{\sum_{d=1}^{D} \left( h_{d,i} - \overline{h_i} \right)^2 \sum_{d=1}^{D} \left( t_{d,j} - \overline{t_j} \right)^2}}$$

$$r_{i,j} = \frac{D \sum_{d=1}^{D} h_{d,i} t_{d,j} - \sum_{d=1}^{D} h_{d,i} \sum_{d=1}^{D} t_{d,j}}{\sqrt{\left( \left( \sum_{d=1}^{D} h_{d,i} \right)^2 - D \sum_{d=1}^{D} h_{d,i}^2 \right) \left( \left( \sum_{d=1}^{D} t_{d,j} \right)^2 - D \sum_{d=1}^{D} t_{d,j}^2 \right)}}$$

#### **CPA**—Picking a subkey

- Use the values of  $r_{i,j}$  to decide which subkey matches our traces most closely.
  - For each subkey i, find the highest value of  $|r_{i,j}|$  .
  - Looking at the maximum values for each subkey, find the highest value of  $|r_{i,j}|$ , the location i of this maximum is our best guess.
- Note that we're only working with absolute values here because we don't care about the sign of the relationship. All we need to know is that a linear correlation exists.

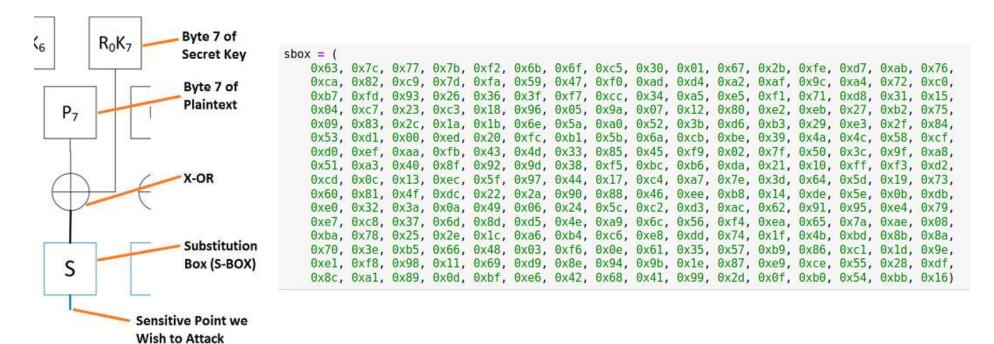
#### CPA—example1(manual CAP AES)

• Capturing traces.

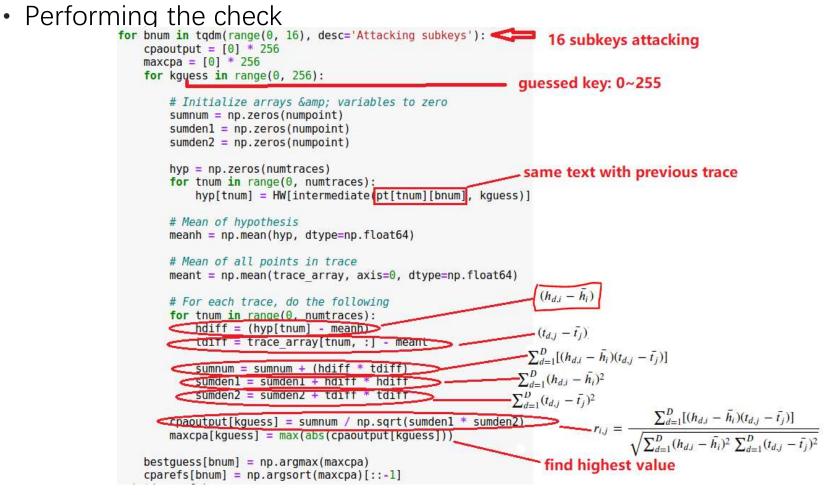
```
from tqdm import thrange
import numpy as np
                              Note: num traces = 50
import time
                                    key is same everytime
ktp = cw.ktp.Basic()
                                    text is random data
traces = []
for i in thrange(num traces, desc='Capturing traces'):
    key, text = ktp.next() # manual creation of a key, text pair can be substituted here
    trace = cw.capture trace(scope, target, text, key)
    if trace is None:
        continue
    traces.append(trace)
#Convert traces to numpy arrays
trace array = np.asarray([trace.wave for trace in traces]) # if you prefer to work with numpy array for number crunce
textin array = np.asarray([trace.textin for trace in traces])
known keys = np.asarray([trace.key for trace in traces]) # for fixed key, these keys are all the same
```

#### CPA—example1(manual CAP AES) cont'd

Review how AES works, attack point at the bottom of the figure.



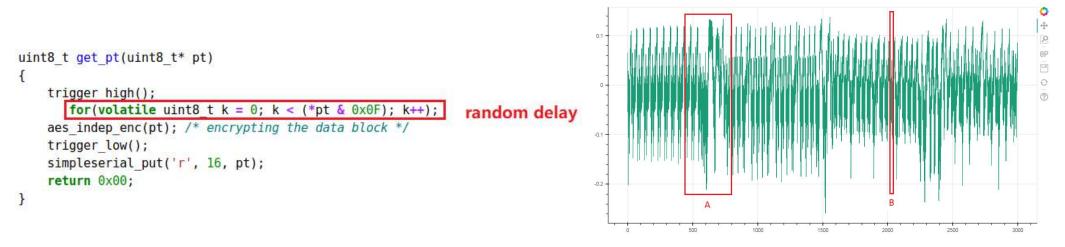
#### CPA—example1(manual CAP AES) cont'd



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#### CPA—example2(resynchronizing jittery AES Power traces)

- Add some random delay before encrypting. This is a countermeasure for CPA.
- The traces are not aligned. How to fix that? Figure out a very unique area, like A in the picture.
  - A window with the "unique" area defined.
  - How far we will shift the window to search for the best match.
- Then, attack like CAP-example1.



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#### **Template Attack—definition**

- Template attacks are a powerful type of side-channel attack. These attacks are a subset of profiling attacks, where an attacker creates a "profile" of a sensitive device and applies this profile to quickly find a victim's secret key.
- To perform a template attack, the attacker must have access to another copy of the protected device that they can fully control.
- To create the template, this may take dozens of thousands of power traces.
- A very small number of traces from the victim to complete the attack.

#### **Template Attack—steps**

- Using a copy of the protected device, record a large number of power traces using many different inputs(plaintexts and keys).
- Create a template of device's operation. This template notes a few "points of interest" in the power traces and a multivariate distribution of the power traces at each point.
- On the victim device, record a small number of power traces. Use multiple plaintexts.(we have no control over the secret key, which is fixed.)
- Apply the template to attack traces. For each subkey, track which value is most likely to the correct subkey. Continue until the key has been recovered.

#### **Template Attack—example**

- Capture and find POI(Points of interest)
  - Capture traces on device fully controlled(Random Key, Random Text, 6000 traces)
  - Separate each trace into a different group according to the resulting hamming weights.
  - Find averages.

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- Find sum of differences
- Find POI(Points of interest)

```
# 5: Find POIs
                                                                    tempMeans = np.zeros((9, len(project template.waves[0])))
 POIs = [1]
oumPOIs = 5 get 5 POIs
                                                                    for i in range(9):
 POIspacing = 5
                                                                        tempMeans[i] = np.average(tempTracesHW[i], 0)
 for i in range(numPOIs):
     # Find the max
                                                                            all of the pairs of traces, subtract them, and add
     nextPOI = tempSumDiff.argmax()
                                                                            them to the sum of differences
     POIs.append(nextPOI)
                                                                         # 4: Find sum of differences
     # Make sure we don't pick a nearby value
                                                                        tempSumDiff = np.zeros(len(project template.waves[0]))
     poiMin = max(0, nextPOI - POIspacing)
                                                                         for i in range(9):
     poiMax = min(nextPOI + POIspacing, len(tempSumDiff))
                                                                             for j in range(i):
     for j in range(poiMin, poiMax):
                                                                                 tempSumDiff += np.abs(tempMeans[i] - tempMeans[j])
         tempSumDiff[j] = 0
 print (POIs)
                                                                         hv.Curve(tempSumDiff).opts(height=600, width=600)
                                               刘维峰(sawyer), liuweifeng168@126.com
1174, 1165, 2522, 2650, 2502
                                  output
```

ktp.fixed key = False #RANDOM KEY in addition to RANDOM TEX

trace = cw.capture trace(scope, target, text, key)

for i in range(len(project template.traces)):

HammingWeight=text[0] ^key[0] every trace, just work for Key(

tempTracesHW[HW].append(project template.waves[i])

for i in thrange(N, desc='Capturing traces'):

N = 6000 # Number of traces

if trace is None: continue

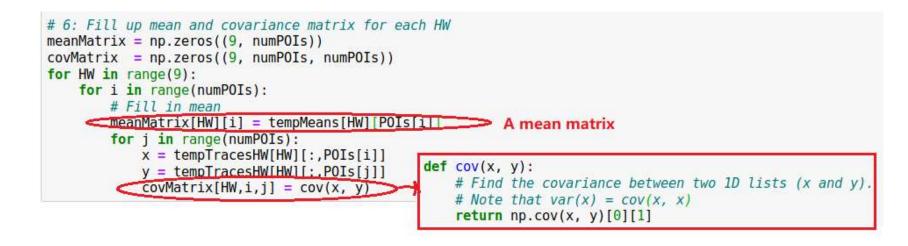
key, text = ktp.next()

HW = tempHW[i]

project.traces.append(trace)

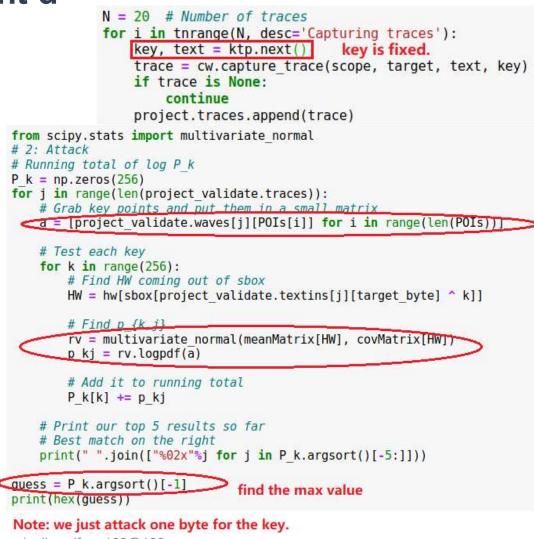
#### Template Attack—example, cont'd

- Generate a Template
  - Build multivariate distributions at each point(POI) for each Hamming weight.
  - A mean matrix(1 x numPOIs) which stores the mean at each POI.
  - A covariance matrix(numPOIs x numPOIs) which stores the variances and covariances between each of the POIs.



#### Template Attack—example, cont'd

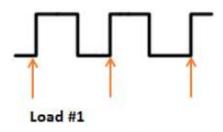
- Applying the Template
  - Capture traces on victim device.
  - Build a multivariate\_normal object using the relevant mean and covariance matrices.
  - Calculate the log of the PDF and add it to the running total.
  - Find the highest value of  $\mathsf{P}^{k}$

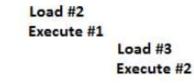


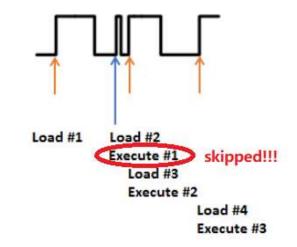
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#### Fault Attack – clock Glitch

- Background
  - Most MCUs have pipeline to speed up the execution process.
  - Modify the clock, the system doesn't have enough time to perform an instruction.

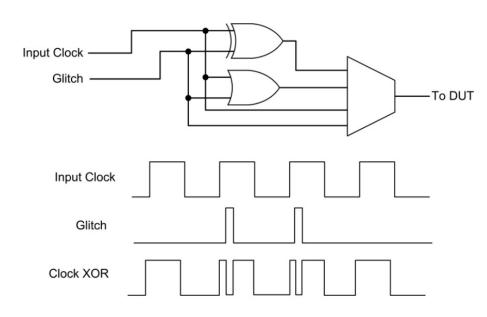


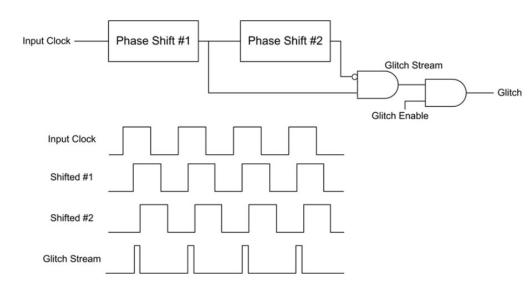




#### Fault Attack - clock Glitch, cont'd

- Glitch Hardware
  - The generation of glitches is done with two variable phase shift modules.
  - Input clock and Glitch XRO, the glitches can be inserted continuously.

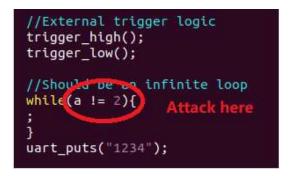




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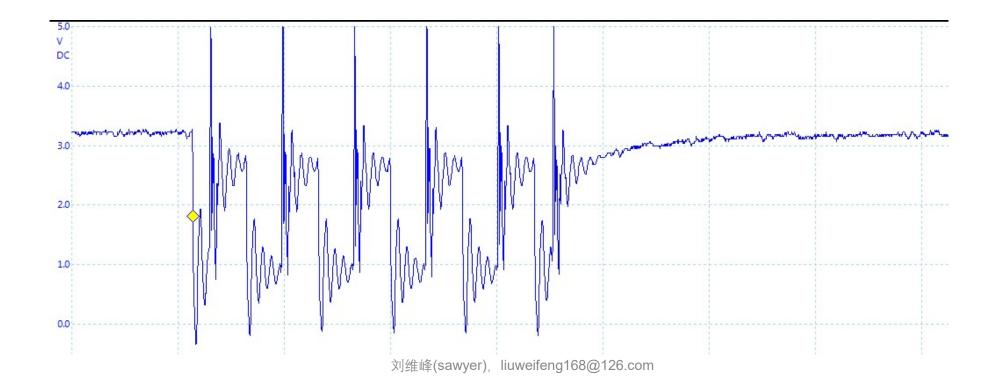
#### Fault Attack – clock Glitch, cont'd

- Example
  - Code. When MCU enters an infinite loop, insert glitch on clock. UART will output "1234" after attacking successfully.



#### **Fault Attack – Power Glitch**

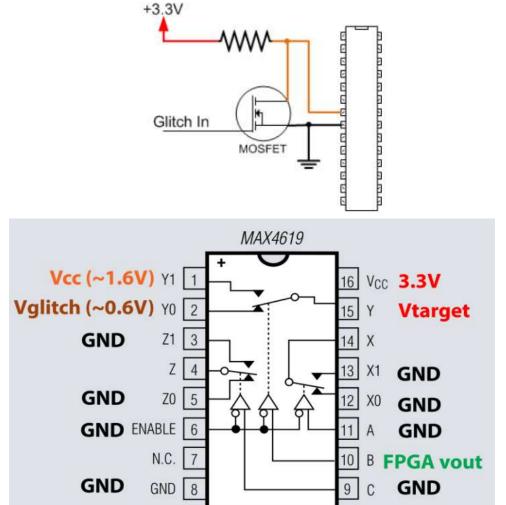
- Background
  - Insert glitch on voltage of device.
  - Can cause a failure to correctly read a memory or havoc with the proper functioning.



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#### Fault Attack – Power Glitch, cont'd

- Hardware
  - A MOSFET to short the power line to GND at specific instances.
  - Use high-speed/CMOS analog switch(e.g MAX4619) to generate glitch.



#### Fault Attack – Power Glitch, cont'd

- Example
  - During normal operation, UART will output "40000 200 200 \$k". The numbers are incorrect, when inserting VCC glitch.

```
void glitch infinite(void)
```

```
{
   char str[64];
   unsigned int k = 0;
   //Declared volatile to avoid optimizing away loop.
   //This also adds lots of SRAM access
   volatile uint16 t i, j;
   volatile uint32 t cnt;
   while(1){
       cnt = 0;
       trigger high();
       trigger low();
       tor(i=0; i<200; i++){</pre>
                                     Add power glitch
            for(j=0; j<200; j++){
                cnt++;
            }
        }
       sprintf(str, "%lu %d %d %d\n", cnt, i, j, k++);
       uart puts(str);
    }
```

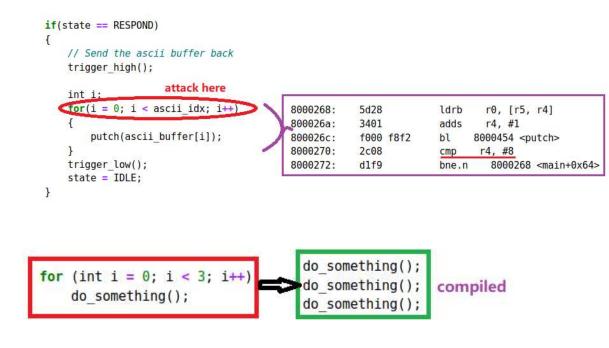
}

### **Fault Attack – Buffer Glitch**

- Background
  - Occur this when more data is put into a fixed-length buffer than the buffer can handle.
  - Do this by well-timed clock or power glitch.
  - Usually result a system crash, or create the opportunity for an attacker to run arbitrary code or manipulate the coding errors

#### Fault Attack – Buffer Glitch, cont'd

- Example
  - The code is compiled, then check the ARM disassembly.
  - The check is whether i == ascii\_idx.
  - Insert power or clock glitch, skip the check.
  - Note: for a small number of repetitions, size-speed tradeoff is skewed towards the latter(Wikipedia page on loop unrolling ).



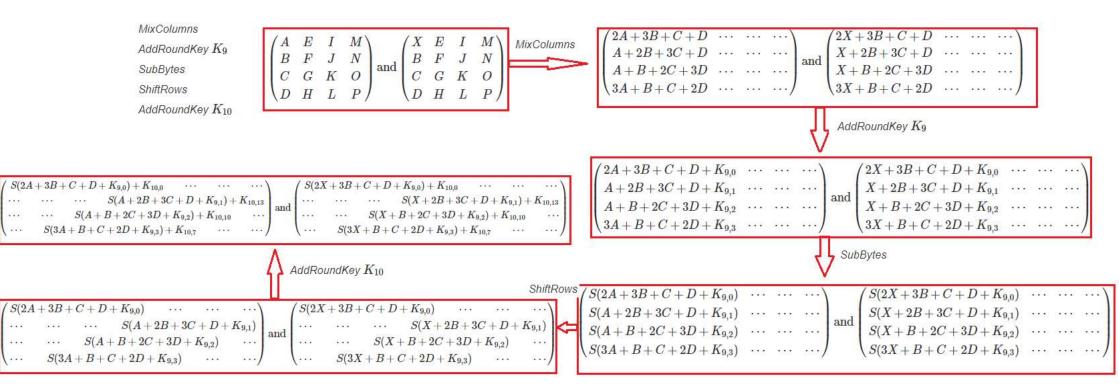
### Fault Attack – Buffer Glitch, cont'd

- Buffer overrun check
  - Insert a guard carriable onto the stack frame for each vulnerable function
  - Tools chains support this function.
    - MCUXpresso IDE: <u>https://mcuoneclipse.com/2019/09/28/stack-canaries-with-gcc-checking-for-stack-overflow-at-runtime/</u>
    - Keil: http://www.keil.com/support/man/docs/armcc/armcc\_chr1359124940593.htm
    - IAR: https://www.iar.com/support/resources/articles/stack-protection-in-iar-embedded-workbench-for-arm/

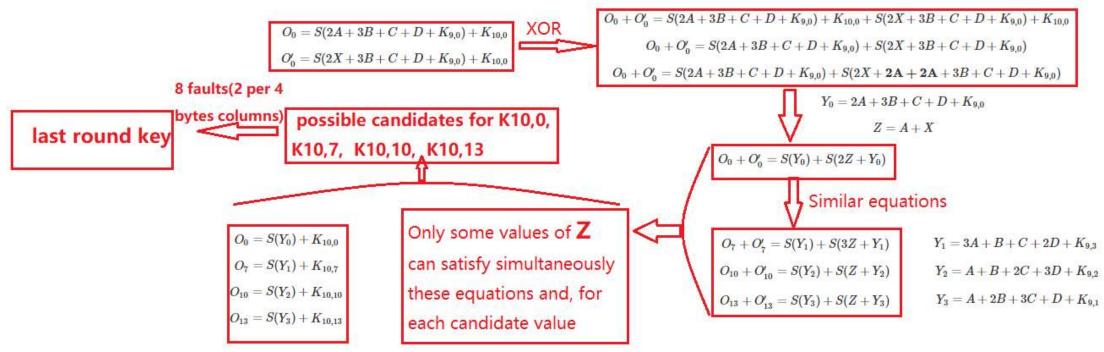
 Note: this is not a 100% check, because it relies on if the program/attacker writes to a memory address on the program's call stack outside of the intended data structure.

- Introduction
  - The basis of attack is that one byte of the state needs to be corrupted somewhere between the two last *MixColumns* operations, which take place in the last 3 rounds.
  - In this situation, the fault will be propagated to exactly 4 bytes of the output.

- Introduction
  - fault the first byte just before the last *MixColumns* operation



- Introduction
  - the following equations for the normal and faulty cases.



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- Example(AES-128, clock glicthing)
  - Attack the 8<sup>th</sup> or 9<sup>th</sup> round.
  - Collecting faulty outputs.
  - Cryptanalysis of the faulty outputs using phoenixAES. And get last round key.
  - Revert the AES keyscheduling and reveal the actual AES key.

### Fault Attack – RSA Fault Attack

• RSA-CRT system

primes p and q, N = pq 
$$i_q = q^{-1} \mod p$$
  
 $n = pq$  and  $\phi = (p-1)(q-1)$ ,  $d_q = e^{-1} \mod (q-1)$   
 $ed \equiv 1 \mod \phi$ ,  $d_p = e^{-1} \mod (p-1)$ 

**Input:** message m, key  $(p, q, d_p, d_q, i_q)$ **Output:** signature  $m^d \in \mathbb{Z}_N \equiv S$ 

$$\begin{split} S_p &= m^{d_p} \mod p \\ S_q &= m^{d_q} \mod q \\ S &= S_q + q \cdot (i_q \cdot (S_p - S_q) \mod p) \\ \textbf{return} \quad (S) \end{split}$$

# Fault Attack – RSA Fault Attack

#### • RSA-CRT fault attack

A fault happens in one of the equations mod p or q. P divides this value, but q does not divide this value. This means that  $\tilde{s}^e - m$  is of the form pk with some integer k,  $gcd(\tilde{s}^e - m, N)$  will give out  $p! \quad q = \sqrt[N]{p}!$ 

$$\begin{cases} s_1 = m^{d_p} \pmod{p} \\ \widetilde{s_2} = m^{d_q} \pmod{q} \end{cases} \implies \widetilde{s} \pmod{pq}$$

$$\begin{cases} \widetilde{s}^e = m \pmod{p} \\ \widetilde{s}^e \neq m \pmod{q} \end{cases} \implies \begin{cases} \widetilde{s}^e - m = 0 \pmod{p} \\ \widetilde{s}^e - m \neq 0 \pmod{q} \end{cases} \implies \begin{cases} p \mid \widetilde{s}^e - m \\ q \nmid \widetilde{s}^e - m \end{cases}$$

## Fault Attack – RSA Fault Attack

- Applying the RSA-CRT attack
  - Insert a glitch
  - Read the signature back
  - Verify that it is correct
  - Extract the PKCS#xx padded hash.
  - $p = gcd(\tilde{s}^e m, N)$
  - q = N/p
  - $ed \equiv 1 \mod \emptyset$ ,  $\emptyset = (p 1)(q 1)$ , **d** is private key.

### TVLA

- Definition
  - TVLA(Test Vector Leakage Assessment) which was proposed at NIST sponsored NIAT workshop 2011, is one of such conformance style testing mechanism which seeks to detect and analyze leakage directly in a device under test.
  - The statistical measurement used with TVLA is welch's t-test for significance of "difference of means" with a threshold of 4.5 standard deviations.
  - TVLA test can be useful for validating your crypto implementation. If the TVLA value is within  $\pm$  4.5, we can claim that the crypto-implementation is secure with high confidence.

## TVLA

- Example
  - See Test Vector Leakage Assessment (TVLA) Derived Test Requirements (DTR) with AES.



### SNR

### • Example

- Capture the trace
- Sort the traces as Hamming Weight and compute means for every HW.
- Remove any groups with zero traces.
- Use hwarray[4] to calculate the noise.

```
N = 1500
for i in tnrange(N, desc='Capturing traces'):
    key, text = ktp.next()
    trace = cw.capture_trace(scope, target, text, key)
    if trace is None:
        continue
    traces.append(trace)
```

<pre>for i in range(0, len(hwarray)):     if len(hwarray[i]) &gt; 0:         inc_list.append(i) hwmean_valid = hwmean[inc_list]</pre>	Remove any groups with zero traces	
<pre>signal_var = np.var(hwmean_valid, noise_var_onehw = np.var(hwarray)</pre>	, axis=0) [4], axis=0)	variance

hwarray = [[], [], [], [], [], [], [], [], []]

#For each byte we are looking at - let's split into multiple groups Sort the trace
for tnum in range(0, len(traces)):
 hw\_of\_byte = HW[intermediate(traces[tnum].textin[bnum], traces[tnum].key[bnum])]
 hwarray[hw of byte].append(traces[tnum].wave)

hwmean = np.zeros((9, npoints))

for i in range(0, 9):
 hwmean[i] = np.mean(hwarray[i], axis=0) compute means for every HW

# CASE ANALYSIS

